“Modeling, Identification and Control of Aerospace Systems”

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Presentation Outline

- Electrohydraulic Control of a Flexible Structure – Bond Graph Approach
- Bond Graph Modeling of a Slewing Structure
- Comparison with Experimental Results
- Conclusions

- Bond-Graphs is a graphical multiphysics dynamic modeling method invented by professor Henry M. Paynter in MIT in 1959;

- BG is a graphycal representation of the flow of energy and power independent of the physical domain;

- BG modeling permits the visualization of the energy channels through the physical systems, where this energy is stored and dissipated by irreversible processes;

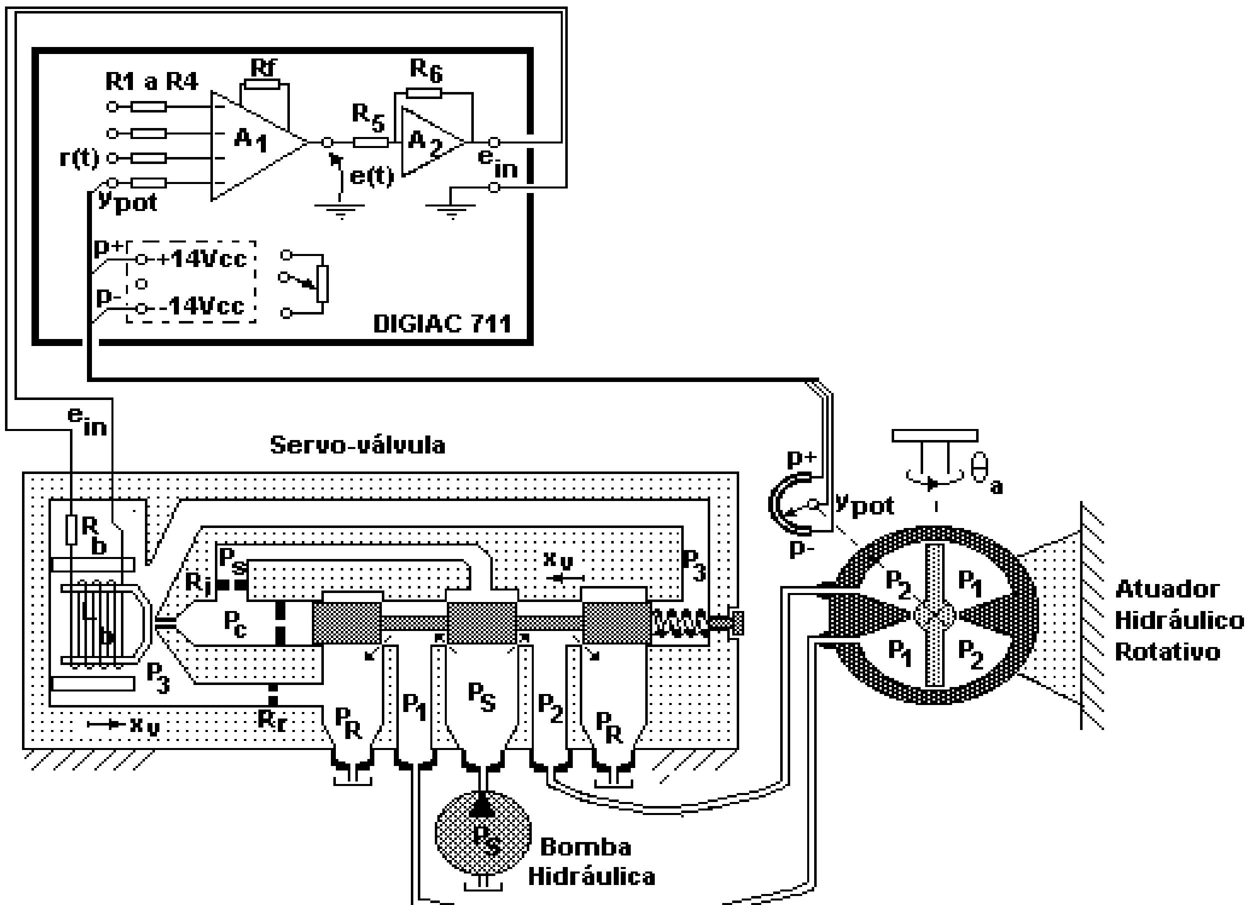
Elements of the BG Language

1 port Elements	- Sources	SE, SF
	- Energy Storage Devices	C_g, I_g
	- Energy Dissipators	R_g
2 portas Elements	- Transformers	TF
	- Gyrators	GY
Elements with two or more ports	- Parallel Junction	0
	- Series Junction	1

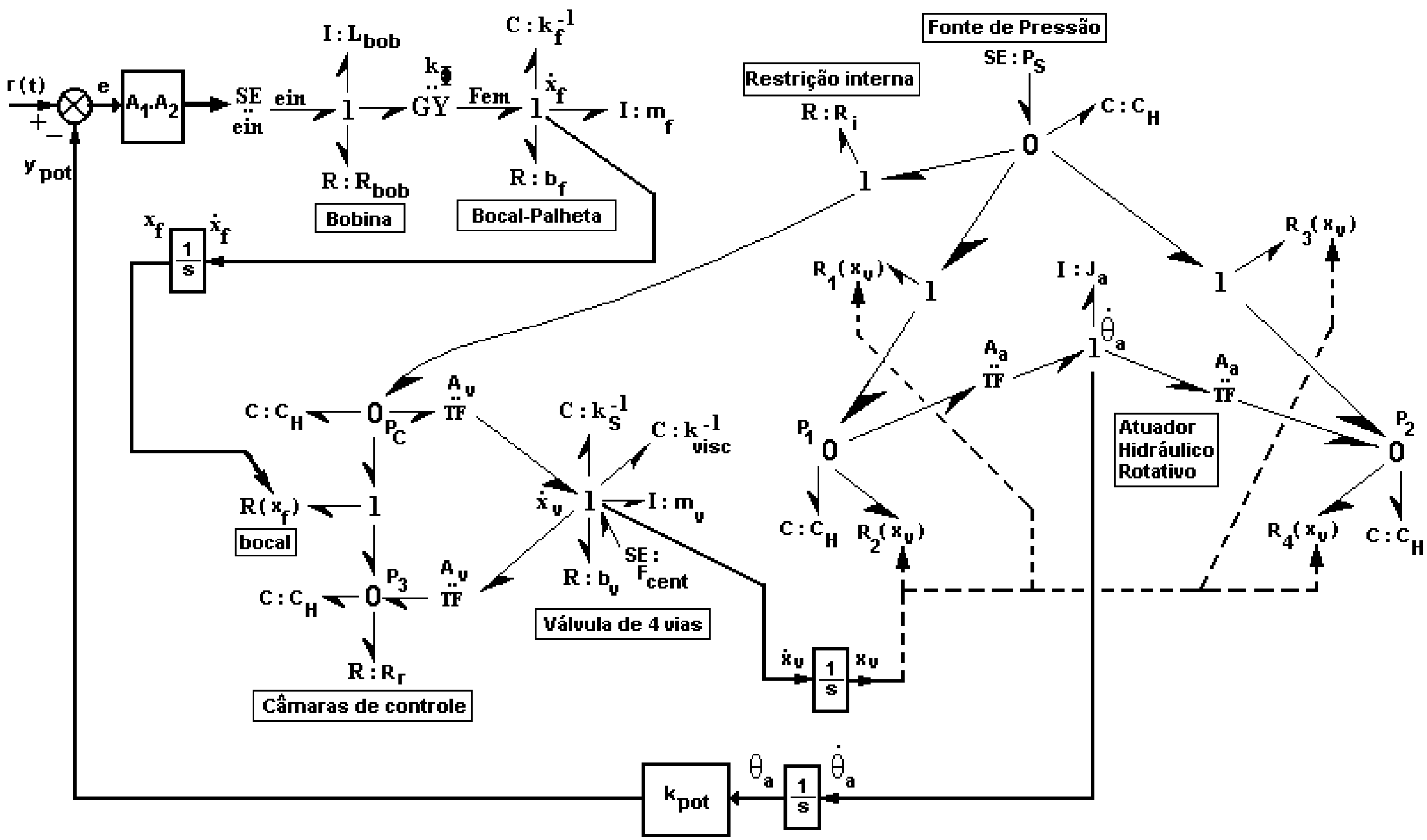
System Description

- Air bearing system
- Central hub
- Flexible structure (Plate)
- Torsion bar
- Electrohydraulic motor
- Position sensor (pot)
- Vibration sensors (acc.)

The Electrohydraulic Plant

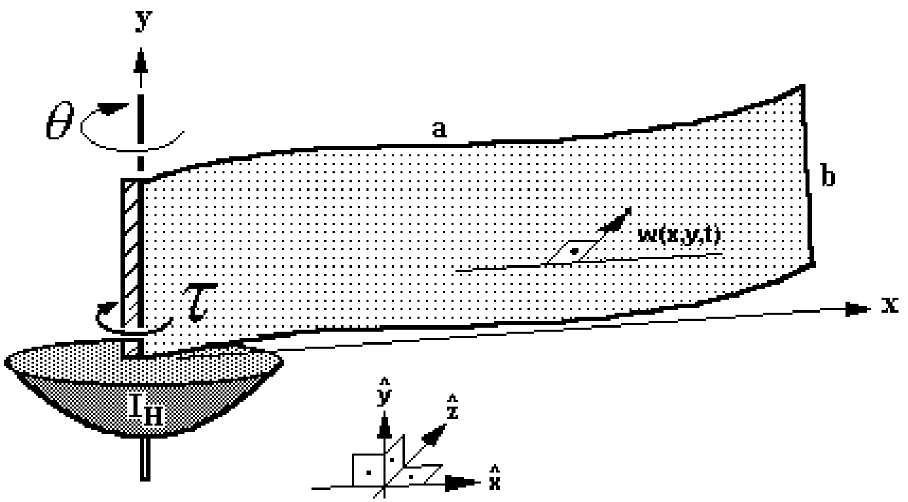


Bond Graph Model – EH System



The Flexible Structure

- Young's modulus, Aluminium
  $E = 6,89 \cdot 10^{10}$ [N/m]



- Mass density, 2795 [kg/m^3]
- Length, a = 1,41 [m]
- Height, b = 46,85 [cm]
- Plate thickness, h= 2,65 [mm]
- Moment of inertia of the hub, I_H = 3,5 [kg.m^2]

Dynamic Model of the Flexible Appendage

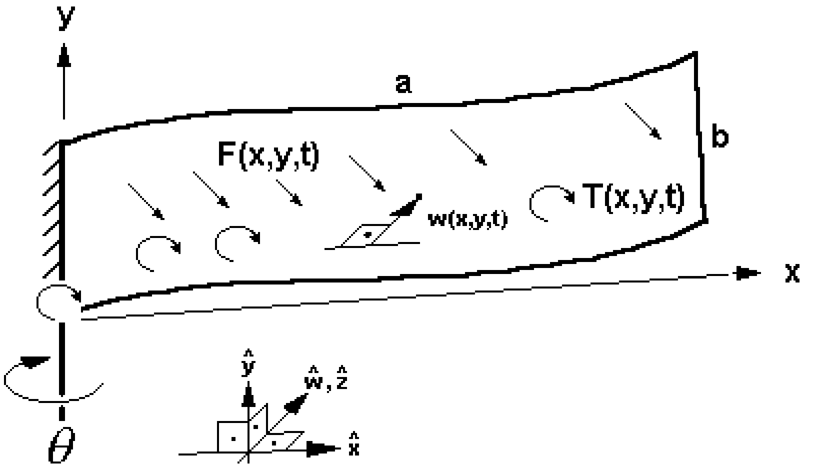
- Hamilton Principle: $$\delta \int_{t_0}^{t_f} \left(K - V_{\text{int}} + W_{nc} \right) dt = 0$$
- Kinetic Energy: $$K = \frac{1}{2} I_H \dot{\theta}^2 + \frac{1}{2} \int_0^b \int_0^a m \left(x\dot{\theta} + \dot{w} \right)^2 dxdy$$
- Potential Energy:

$$V_{\text{int}} = \frac{1}{2} \int_0^b \int_0^a D \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1-\nu) \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \right\} dxdy$$

Equation of Motion

- Rigid body motion, $\theta(t)$
- Elastic displacement, w(x,y,t)

$$\begin{cases} (I_H + I_P)\ddot{\theta} + \int_0^b \int_0^a m\, x \dfrac{\partial^2 w}{\partial t^2}\, dx\, dy = \tau \\ D\nabla^4 w + m \dfrac{\partial^2 w}{\partial t^2} + m\, x\, \ddot{\theta} = 0 \end{cases}$$



System Discretization: Assumed Modes

- Total displacement : $$z(x,y,t)=w(x,y,t)+x\theta$$
- Modal Expansion : $$z(x,y,t)=\sum_{n=1}^{R}\sum_{m=1}^{S}\phi_{nx}(x)\,\phi_{my}(y)\,\eta_{nm}(t)$$
- Euler-Bernoulli functions: pinned-free in the x-direction (bending) and free-free in the y-direction (torsion)

Modal Equations

- **Mass Matrix:**

$$\underline{\underline{M}}_{ij} = \int_0^a \int_0^b \phi_{px}\phi_{qy} m_j \, dxdy$$

$$m_j = m\phi_{nx}\phi_{my}$$

- **Stiffness Matrix:**

$$\underline{\underline{K}}_{ij} = \int_0^a \int_0^b \phi_{px}\phi_{qy} k_j \, dxdy$$

$$k_j = D\left(\phi_{nx}^{iv}\phi_{my} + 2\phi_{nx}^{ii}\phi_{my}^{ii} + \phi_{nx}\phi_{my}^{iv}\right)$$

- **Modal Equation:**

$$\underline{\underline{M}}\ddot{\eta} + \underline{\underline{K}}\eta = \underline{0}$$

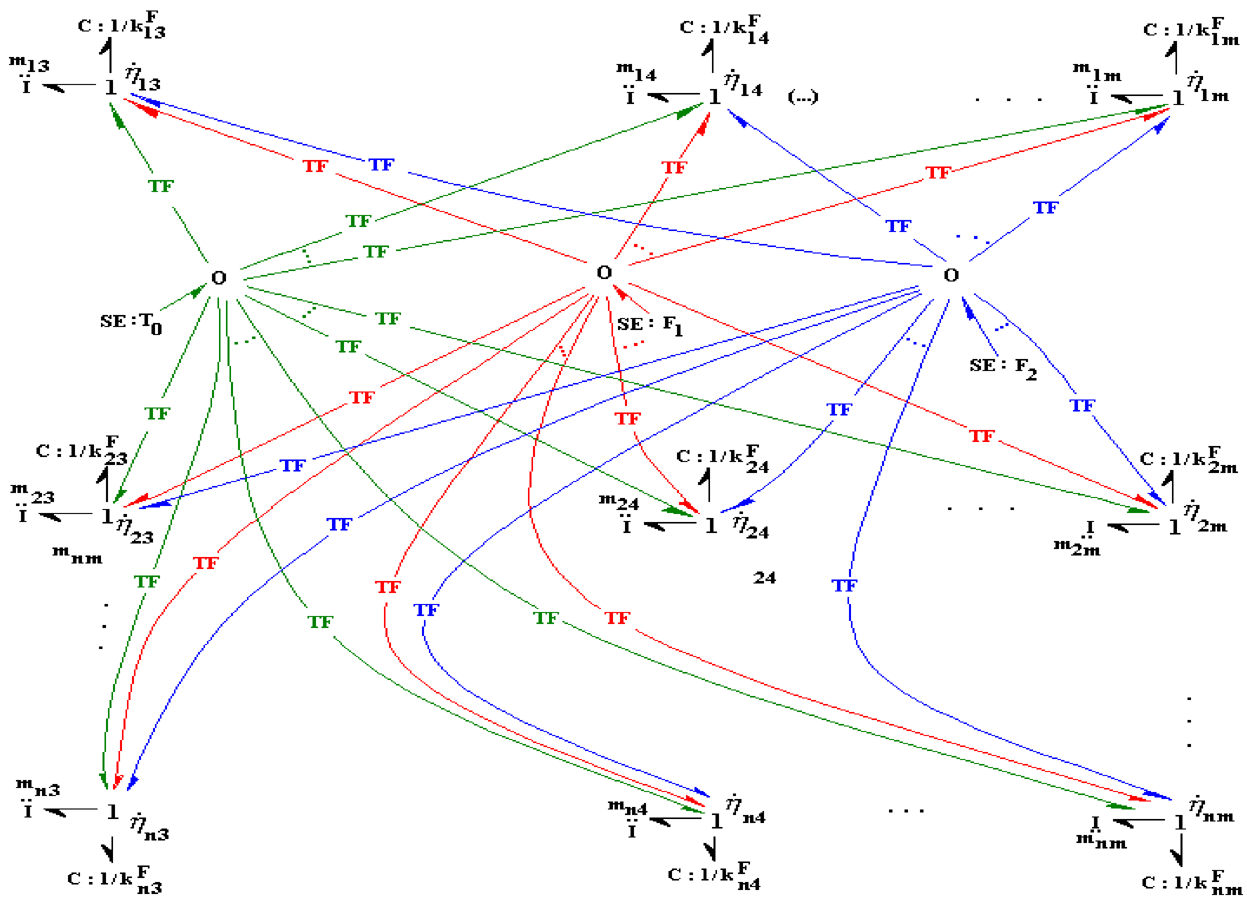
Modal Forces and Flexible Modes

- **Controlled moment :** $T(x,y,t)=T_0\delta^{'}(y)$
- **External efforts :** $F(x,y,t)=F_1\delta(x-x_1)\delta(y-y_1)+F_2\delta(x-x_2)\delta(y-y_2)$
- **Flexible Modes :**

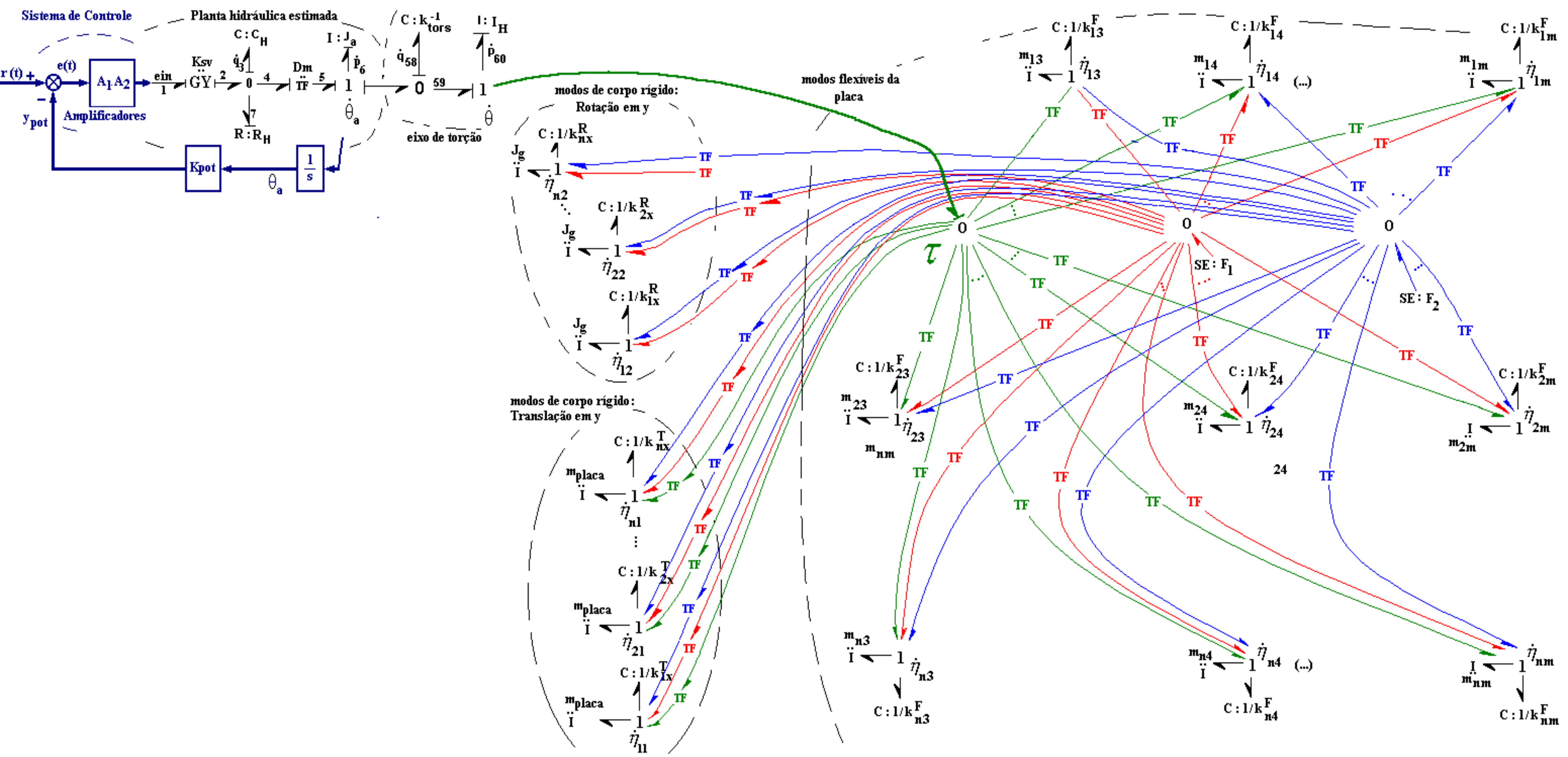
$$m_{nm}\ddot{\eta}_{nm}+k_{nm}^{F}\eta_{nm}=F_1.\phi_{qy}(y_1)\int_0^a\phi_{nx}dx+F_2.\phi_{my}(y_2)\int_0^a\phi_{nx}dx+T_0\phi_{nx}^{'}(0)\int_0^b\phi_{my}\,dy$$

$$k_{nm}^{F}=D\left[b\int_0^a\phi_{nx}^{iv}\,\phi_{nx}dx+2\int_0^a\phi_{nx}^{ii}\phi_{nx}\,dx\int_0^b\phi_{my}^{ii}\phi_{my}dy+a\int_0^b\phi_{my}^{iv}\,\phi_{my}dy\right]$$

Bond-Graph Representation of the Flexible Plate



Bond-Graph of the Controlled Flexible Structure with Electro Hydraulic Actuation



Bond-Graph of an Aerospace Flexible Structure with Hydraulic Actuation

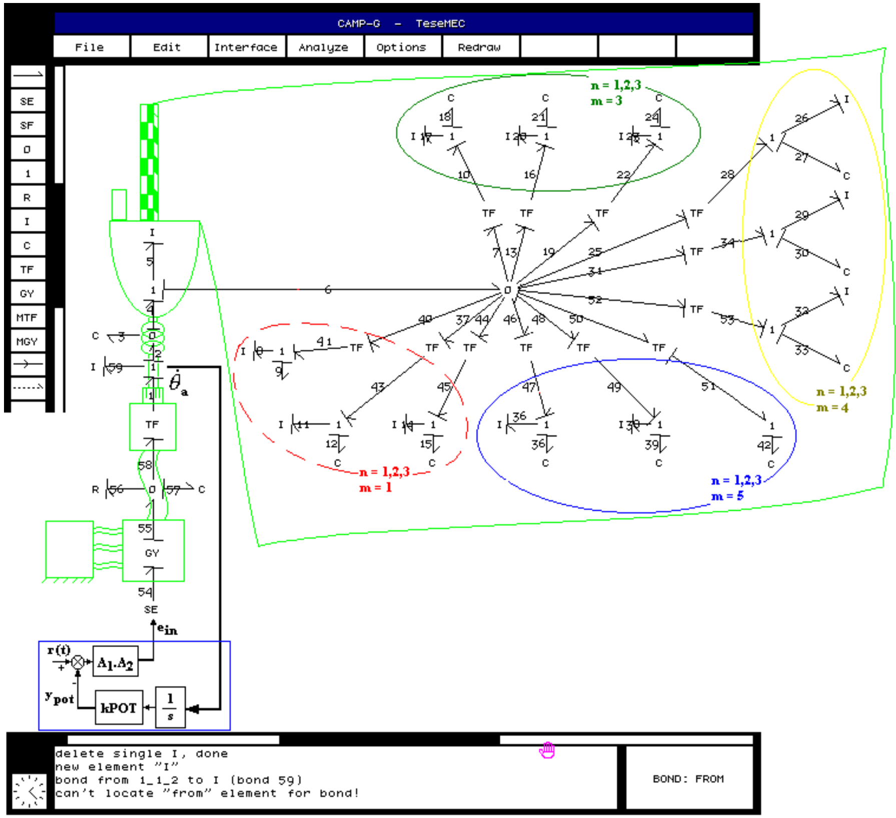
- **The BG description of the slewing link is coupled to the low-order equivalent system for the hydraulic actuation system;**

- **The colour lines (R,G,B) depict graphically the influence of the control torque at the hub and the coupling forces and moments between the various flexible modes of the system;**

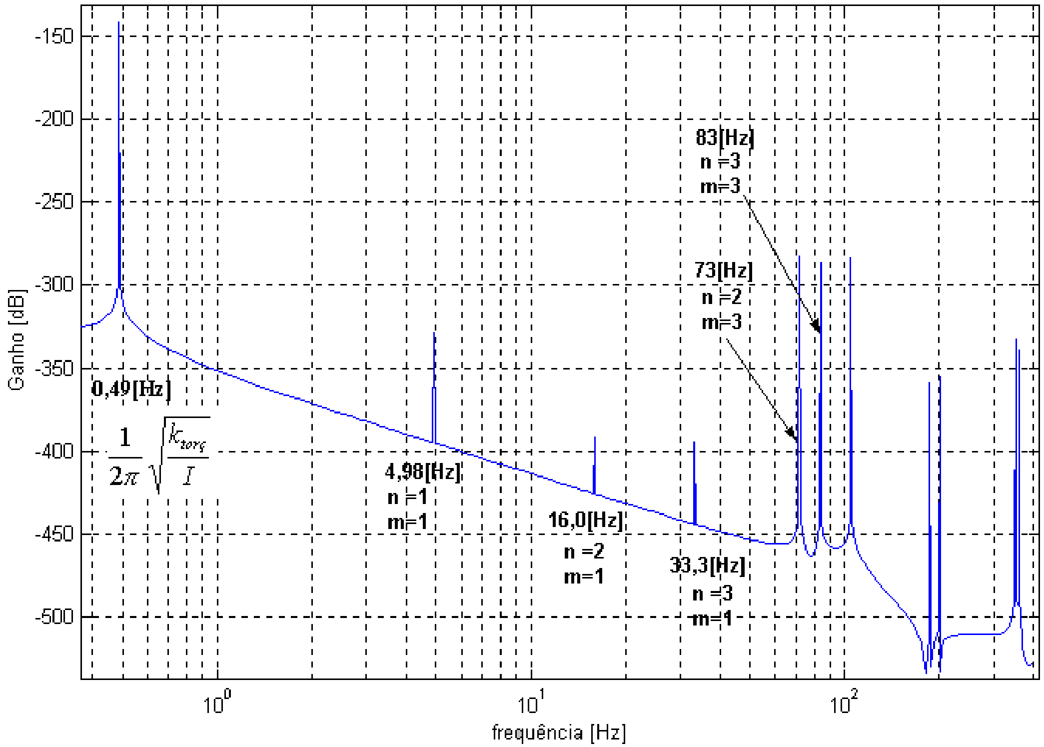
- **This is a unique way to visualize a complex dynamic system that has rigid body modes coupled to flexural and torsional energy in the system, and the low order graphical description of the electro hydraulic servo-actuation system.**

Bond Graph Simulation

- Bond-Graph Simulation with CAMP-G;
- Normal modes estimation with theoretical FRFs
- The simulation results are validated by comparing the computed FRF, taking the response of an accelerometer fixed at the free edge of the plate (x=a and y=b/2)



Modal Response Calculation

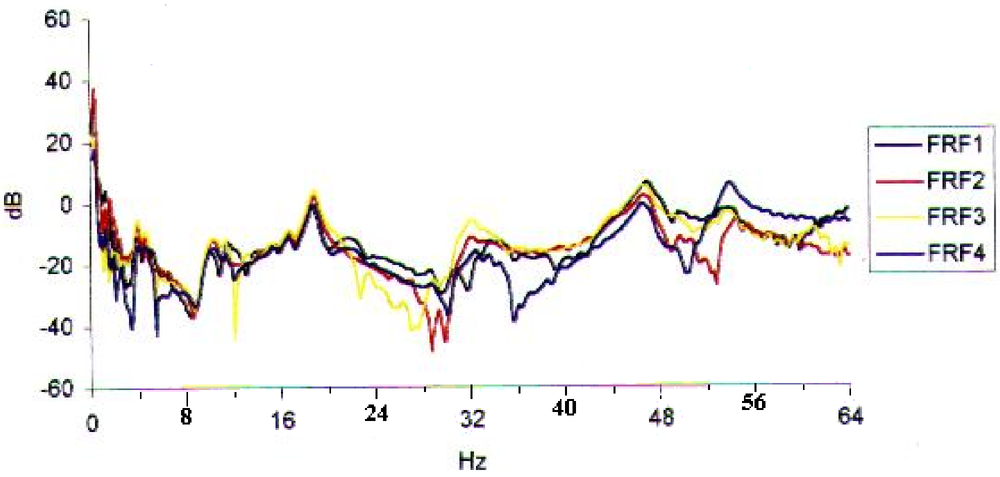


Frequency Response Function (FRF) obtained with the linearized model described in the Camp-G environment.

Comparison with Experimental Results

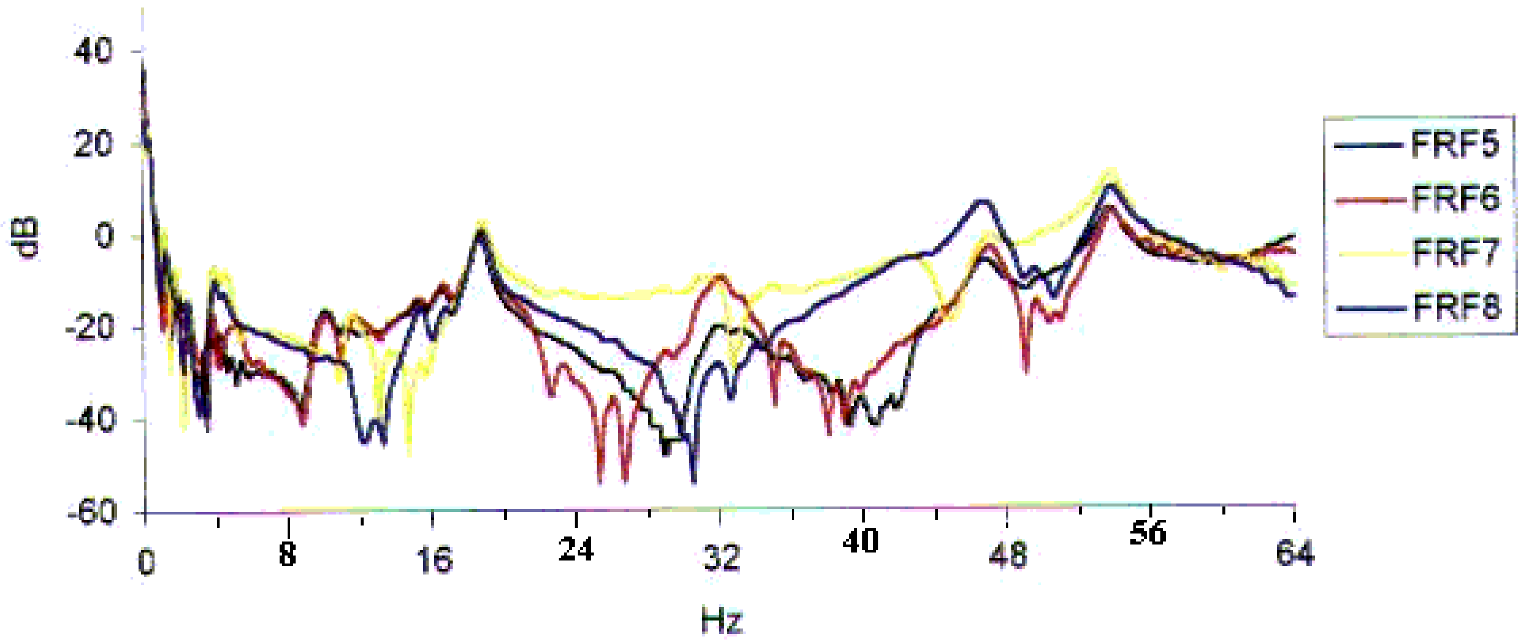
- **To validate the BG model, a comparison is made between the theoretical-FRF and the experimental-FRF estimated by modal testing as shown in the figure;**

- **The plate modes were excited with a shaker, with the central hub in a fixed position.**



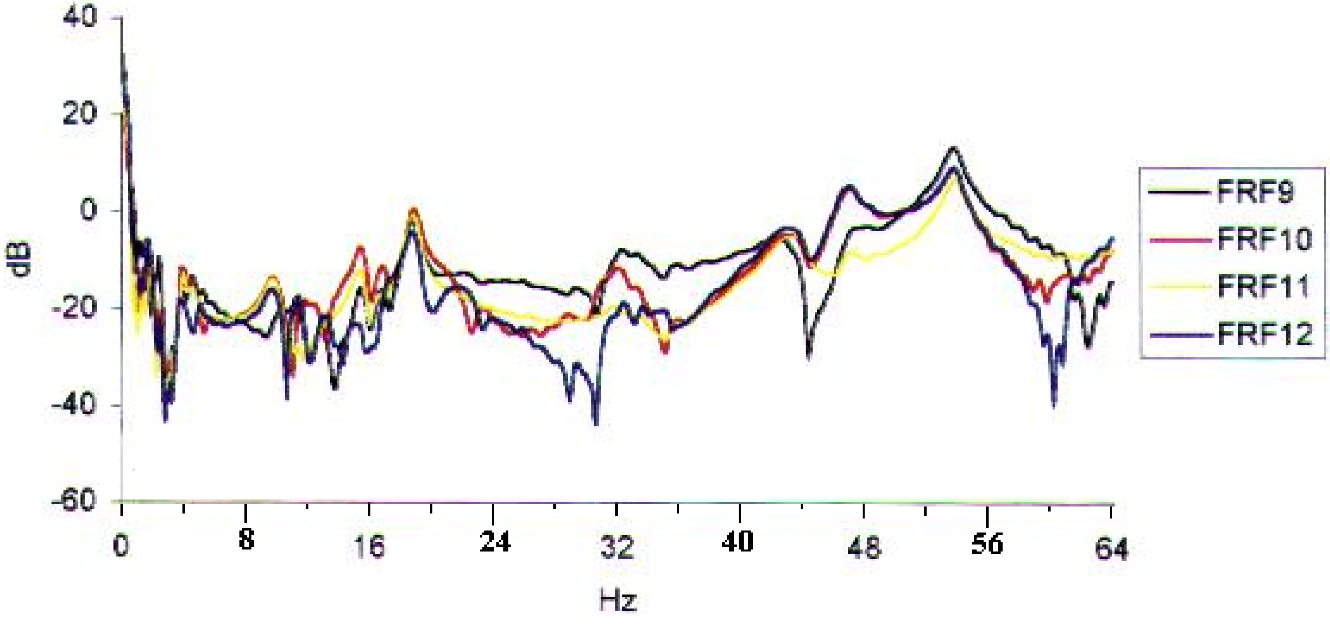
Experimentally Identified FRF

- **Experimental FRFs of the Flexible Plate, accelerometers 5, 6, 7 and 8.**



Experimental FRF

- Experimental FRFs of the Fixed Plate, accelerometers 9, 10, 11 and 12.



Resonance Frequencies (Hz)

Modal Equations	CAMP-G Bond-Graph Model	Experimental Modal Testing
4,41	4,98	5,0
15,9	16,0	19,0
33,2	33,3	33,0
66,8	66,0	*Out of Freq.Band
73,5	73	*
86,1	84	*

Conclusions

- This work presented a BG model of a slewing flexible plate controlled by a hydraulic servo control system;
- The BG model reproduced the principal characteristics of the multi-domain dynamical system, with the advantage of providing a direct visualization of the interaction of principal dynamic effects and its coupling characteristics;

Conclusions

- The analytical frequencies, derived from the BG model, at 4.9; 16.9 and 33.2 [Hz] can be associated with the experimentally observed flexible modes (n, m) = (1,1), (2,1) and (3,1), respectively;
- Other BG predicted modes at 73.5 and 86.1[Hz] are coherent with the modes (n,m) = (2,3) and (3,3), respectively;
- The frequency at 66.0 [Hz] is associated with flexible mode (n,m)=(1,3).